

VALITEST webinar series and training activities

Using estimates gained during test performance studies

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Fera



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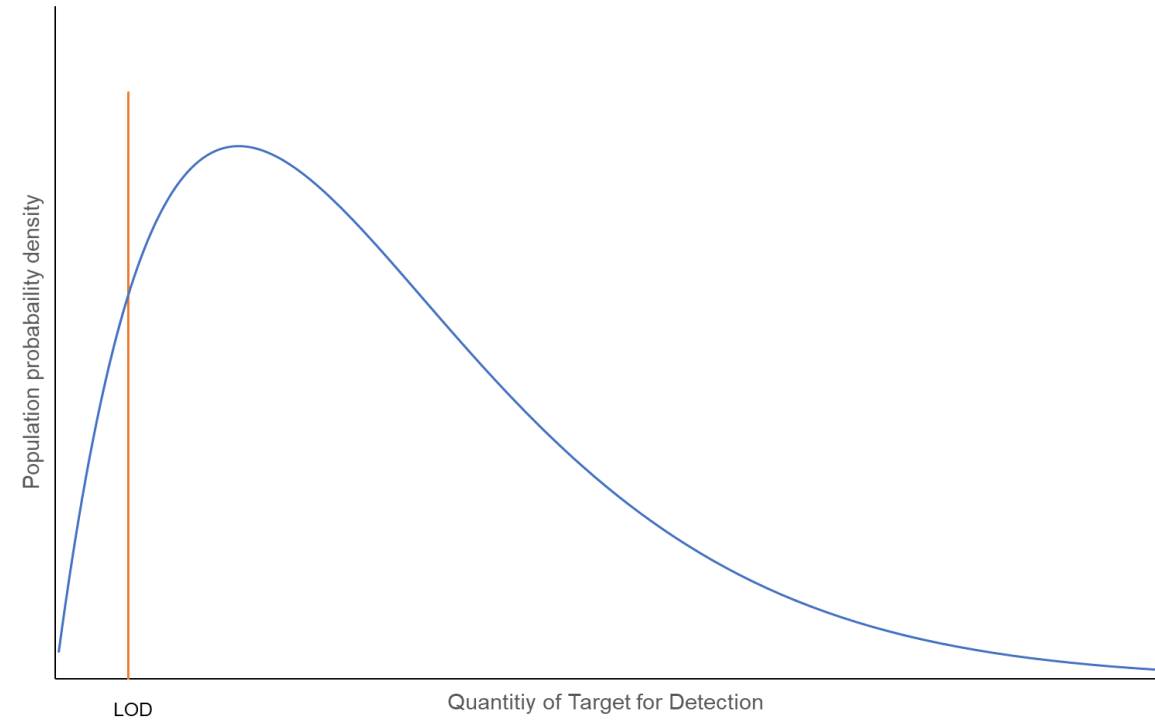
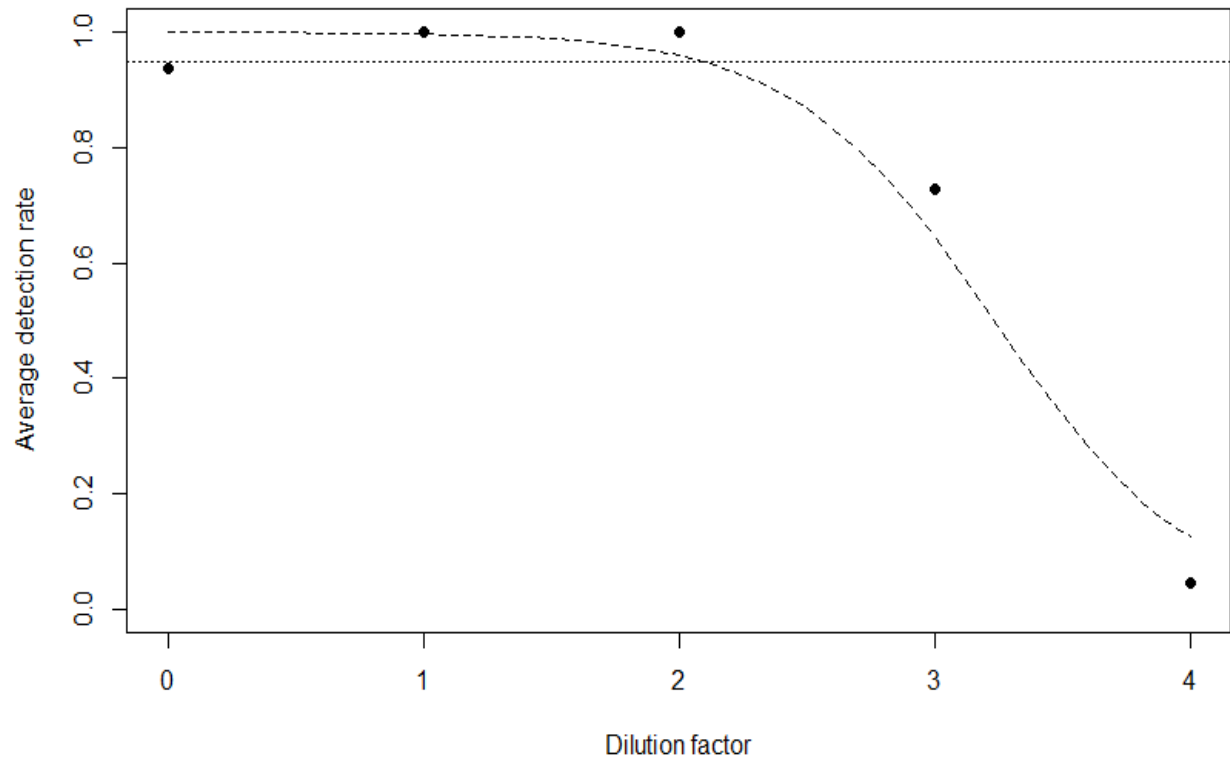
TPS estimates and fitness for purpose

- We can use TPS results to predict how a method is likely to perform
- Combined with the cost of testing we can predict fitness for purpose
- This introduces the prediction of fitness for purpose for a number of types of scenario that should cover most cases

Fitness for PURPOSE

- Early detection: detection at lowest prevalence
- Delineation of outbreak: detection at lowest prevalence
- “Eradication”: all negative results are correct
- Monitoring presence: most accurate estimate of prevalence

Information from TPSs



Information from TPSs

	Method 1	Method 2	Method 3
Transferable	✓	✓	✓
Robust	✓	✓	✓
Timely	✓	✓	✗
Diagnostic sensitivity	Se_1	Se_2	Se_3
Diagnostic Specificity	Sp_1	Sp_2	Sp_3
Cost per test	L_1	L_2	L_3

Early detection

- A pathogen may be present.
- If it is present it is still at a low prevalence but growing exponentially with rate R per time
- We have a perfect test with 100% diagnostic specificity and 100% diagnostic sensitivity; we have a test budget of B per time
- What might the highest prevalence (with 95% confidence) be when we first detect the pathogen using a test with cost L

$$N = \frac{B}{L} \quad prev_{95} = \frac{3.R}{N}$$

Early detection

- A pathogen may be present.
- If it is present it is still at a low prevalence but growing exponentially with rate R per time
- We have a **perfect** test with 100% diagnostic specificity and a **diagnostic sensitivity of Se** ; we have a test budget of B per time
- What might the highest prevalence (with 95% confidence) be when we first detect the pathogen using a test with cost L

$$N = \frac{B}{L} \quad prev_{95} = \frac{3.R}{N.Se}$$

Early detection

- A pathogen may be present.
- If it is present it is still at a low prevalence but growing exponentially with rate R per time
- We have a test with a diagnostic specificity Sp and a diagnostic sensitivity of Se ; we have a test budget of B per time. If we get a false positive there is a cost F to resolve it
- What might the highest prevalence (with 95% confidence) be when we first detect the pathogen using a test with cost L

$$N = \frac{B}{L+F.(1-Sp)} \quad prev_{95} = \frac{3.R}{N.Se}$$

Early detection

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- What might the highest prevalence (with 95% confidence) be when we first detect the pathogen using a test with cost L

$$N = \frac{B}{L+F.(1-Sp)}$$

$$prev_{95} = \frac{3.R}{N.Se}$$

$$N_{eff} = \frac{B.Se}{L+F.(1-Sp)}$$

Delineation of an outbreak

- A pathogen may be present in an area.
- If it is present it is still at a low prevalence ~~but growing exponentially with rate R per time~~
- We have a test with a diagnostic specificity Sp and a diagnostic sensitivity of Se ; we have a test budget of B per area. If we get a false positive there is a cost F to resolve it
- What might the highest prevalence (with 95% confidence) be in an area in which we don't detect the pathogen using a test with cost L

$$N = \frac{B}{L+F.(1-Sp)}$$

$$prev_{95} = \frac{3}{N.Se}$$

$$N_{eff} = \frac{B.Se}{L+F.(1-Sp)}$$

Delineation of an outbreak

- A pathogen may be present in an area.
- If it is present it is at a low prevalence.
- We have a test with a diagnostic specificity Sp and a diagnostic sensitivity of Se ; we have a test budget of B per area. If we get a false positive there is a cost F to resolve it
- What might the highest prevalence (with 95% confidence) be in an area in which we don't detect the pathogen using a test with cost L

$$N = \frac{B}{L+F.(1-Sp)}$$

$$prev_{95} = \frac{3}{N.Se}$$

$$N_{eff} = \frac{B.Se}{L+F.(1-Sp)}$$

Eradication

- Applicable where there a small number of valuable plants at risk.
- We need to remove every infected individual
- It is nice if we can also save some uninfected individuals
- Untested plants are assumed infected
- We need high confidence about each negative test result; a sufficiently high negative predicative value

$$NPV = \frac{\text{expected number true negative}}{\text{expected number true negative} + \text{expected number false negative}}$$

$$\frac{\text{expected number true negative}}{\text{expected number false negative}} = \frac{1 - \text{prev}}{\text{prev}} \times \frac{Sp}{1 - Se}$$

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Eradication II

- Applicable where there a small number of valuable plants at risk.
- We need to remove every infected individual
- It is nice if we can also save some uninfected individuals
- Untested plants are assumed infected
- We need high confidence about ALL negative test results; a sufficiently high negative predicative value for zero infected among all negative test results

$$NPV_{ALL} = \left(\frac{\text{number true neg.}}{\text{number true neg.} + \text{number false neg}} \right)^{N \text{ neg. results}}$$

Probability of NO false negatives where all results are negative

- Upper limit (95% confidence) for prevalence = $3/N \cdot Se$
- HENCE!
- $NPV_{ALL} > e^{(3-3/Se)}$ for $N > 30$
- Approximation: maximum (95% confidence) probability of there being ANY false positives among all of the tested plants giving a negative result is 3 x the false positive rate of the test: $NPV_{ALL} > 1 - 3 \cdot (1 - Se)$ for $N > 30$ and $Se > 0.85$ IF ALL TESTS ARE NEGATIVE

Monitoring prevalence

- We want to estimate the prevalence in a population
- Assume we can sample at random!
- We use a perfect method to detect a pathogen in each plant: 100% diagnostic specificity, 100% diagnostic sensitivity
- We detect x positive plants out of n tested

x positive out of n tested: perfect method

Result	Estimate	Lower 95% confidence	Upper 95% confidence
$0 < x < n$	x/n	$((x+1.64)/(n+2.71)) - 1.64 \cdot \sqrt{((x+1.64)/(n+2.71)) \cdot (1 - ((x+1.64)/(n+2.71))) / (n+2.71)}$	$((x+1.64)/(n+2.71)) + 1.64 \cdot \sqrt{((x+1.64)/(n+2.71)) \cdot (1 - ((x+1.64)/(n+2.71))) / (n+2.71)}$

Agresti-Coull method (for $n > 30$)

Brown LD, Cai TT, DasGupta A (2001) Interval Estimation for a Binomial Proportion, *Statistical Science*, **16 (2)** 101 - 133

x positive out of n tested: perfect method

Result	Estimate	Lower 95% confidence	Upper 95% confidence
x=0	x/n	0	$1-0.05^{(1/n)}$
0<x<n	x/n	$BETAINV(0.05,x+0.5,n-x+0.5)$	$BETAINV(0.95,x+0.5,n-x+0.5)$
x=n	x/n	$0.05^{(1/n)}$	1

Modified Jeffreys method (for any n)

Brown LD, Cai TT, DasGupta A (2001) Interval Estimation for a Binomial Proportion, *Statistical Science*, **16 (2)** 101 - 133

x positive out of n tested: perfect method

Outcome	Estimate	95% Confidence interval	
		Lower	Upper
x=0	x/n	0	$1-0.05^{(1/n)}$
0<x<n	x/n	$BETAINV(0.025,x+0.5,n-x+0.5)$	$BETAINV(0.975,x+0.5,n-x+0.5)$
x=n	x/n	$0.05^{(1/n)}$	1

Upper limit for prevalence (95% confidence) if we don't find any positives

$$1 - 0.05^{\frac{1}{n}} \approx \frac{3}{n} \text{ for } n \text{ more than } 30$$

x positive out of n tested: sensitivity =Se,
specificity=Sp

Model:

- x is a random observation from a binomial distribution with size n and probability p
- $p = prev \cdot Se + (1 - prev) \cdot (1 - Sp)$
- prev is described by a beta distribution, initially a flat uniform distribution (with shape parameters (1,1))
- Prevalence =...the prevalence package in R can do all of the hard work
- Speybroeck N, Devleesschauwer B, Joseph L, Berkvens D (2013)
Misclassification errors in prevalence estimation: Bayesian handling with care. Int J Public Health 58:791-795

Examples: early detection; delineation; eradication; monitoring prevalence

- Three validated methods

Test	Cost	Performance	
		Sensitivity	Specificity
LFD	10	50%	99.5%
ELISA	30	95%	99.7%
DNA	100	99%	100%

- Cost associated with resolving a false positive in early detection and delineation scenarios equivalent to five DNA tests.

Early detection

- Budget (B) = 10 000
- Growth rate (R) = 0.693 doubling every budget period

Test	Cost (L)	Performance		N	N _{eff}	P _{rev95}
		Sensitivity (Se)	Specificity (Sp)			
LFD	10	50%	99.5%	800.0	400.0	0.52%
ELISA	30	90%	99.7%	317.5	285.7	0.73%
DNA	100	99%	100%	100.0	99.0	2.10%

$$N = \frac{B}{L+F.(1-Sp)} \quad prev_{95} = \frac{3.R}{N.Se} \quad N_{eff} = \frac{B.Se}{L+F.(1-Sp)}$$

Outbreak delineation

- Budget (B) = 1000 per area

Test	Cost (L)	Performance		N	N _{eff}	P _{rev95}
		Sensitivity (Se)	Specificity (Sp)			
LFD	10	50%	99.5%	80.0	40.0	7.5%
ELISA	30	90%	99.7%	31.7	28.6	10.3%
DNA	100	99%	100%	10.0	9.9	30.3%

$$N = \frac{B}{L+F.(1-Sp)}$$

$$prev_{95} = \frac{3}{N.Se}$$

$$N_{eff} = \frac{B.Se}{L+F.(1-Sp)}$$

Eradication

- Test budget (B) = 10 000
- + new options: a) just do nothing; b) burn everything

Test	Cost (L)	Performance	
		Sensitivity (Se)	Specificity (Sp)
NOTHING	0	0%	100%
LFD	10	50%	99.5%
ELISA	30	90%	99.7%
DNA	100	99%	100%
BURN	0	100%	0%

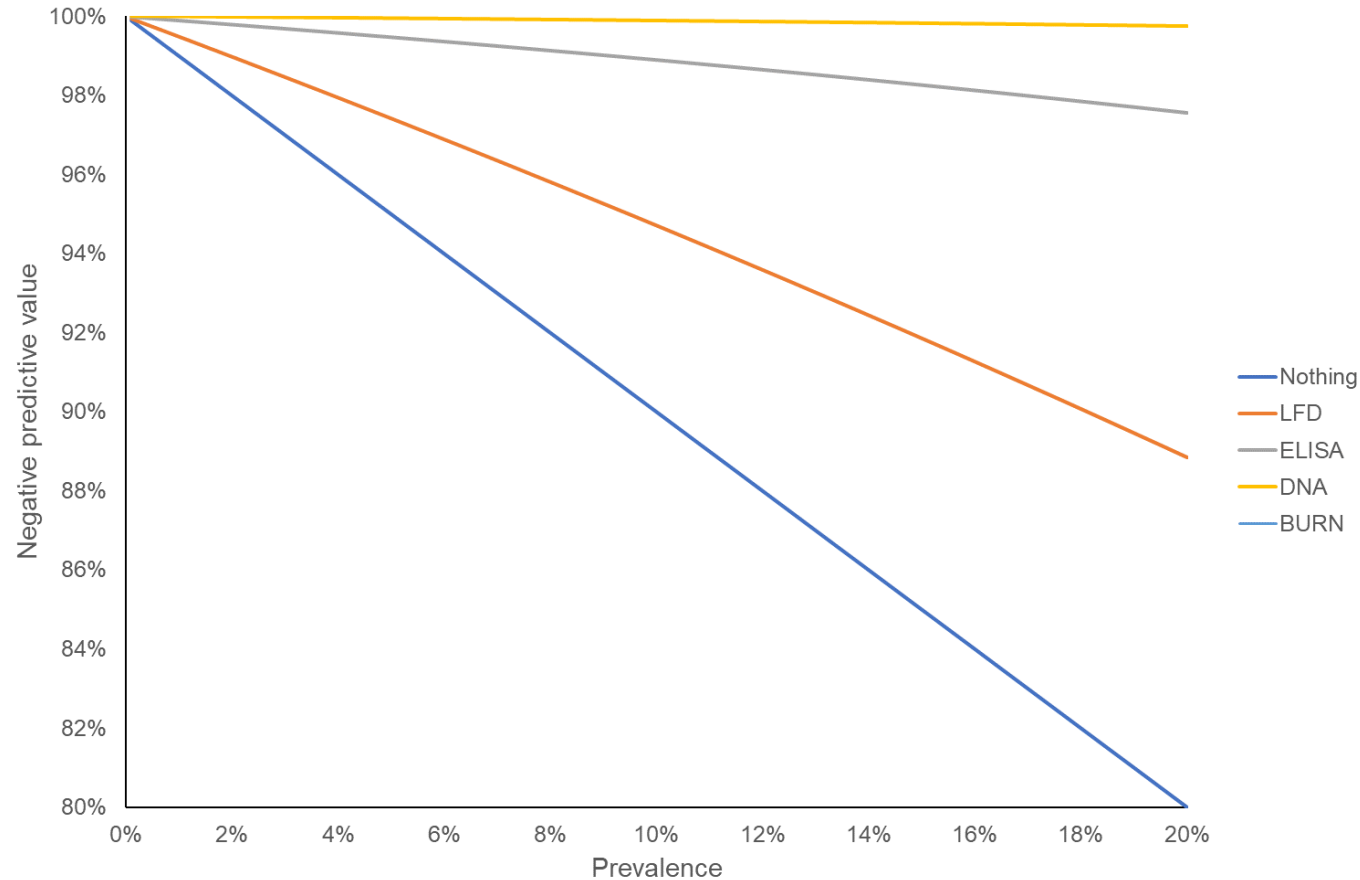
Eradication

- Test budget (B) = 10 000
- + new options: a) just do nothing; b) burn everything

Test	Cost (L)	Performance		Number	NP ratio
		Sensitivity (Se)	Specificity (Sp)		
NOTHING	0	0%	100%	∞	1
LFD	10	50%	99.5%	800.0	1.99
ELISA	30	90%	99.7%	317.5	9.97
DNA	100	99%	100%	100.0	100
BURN	0	100%	0%	∞	NA

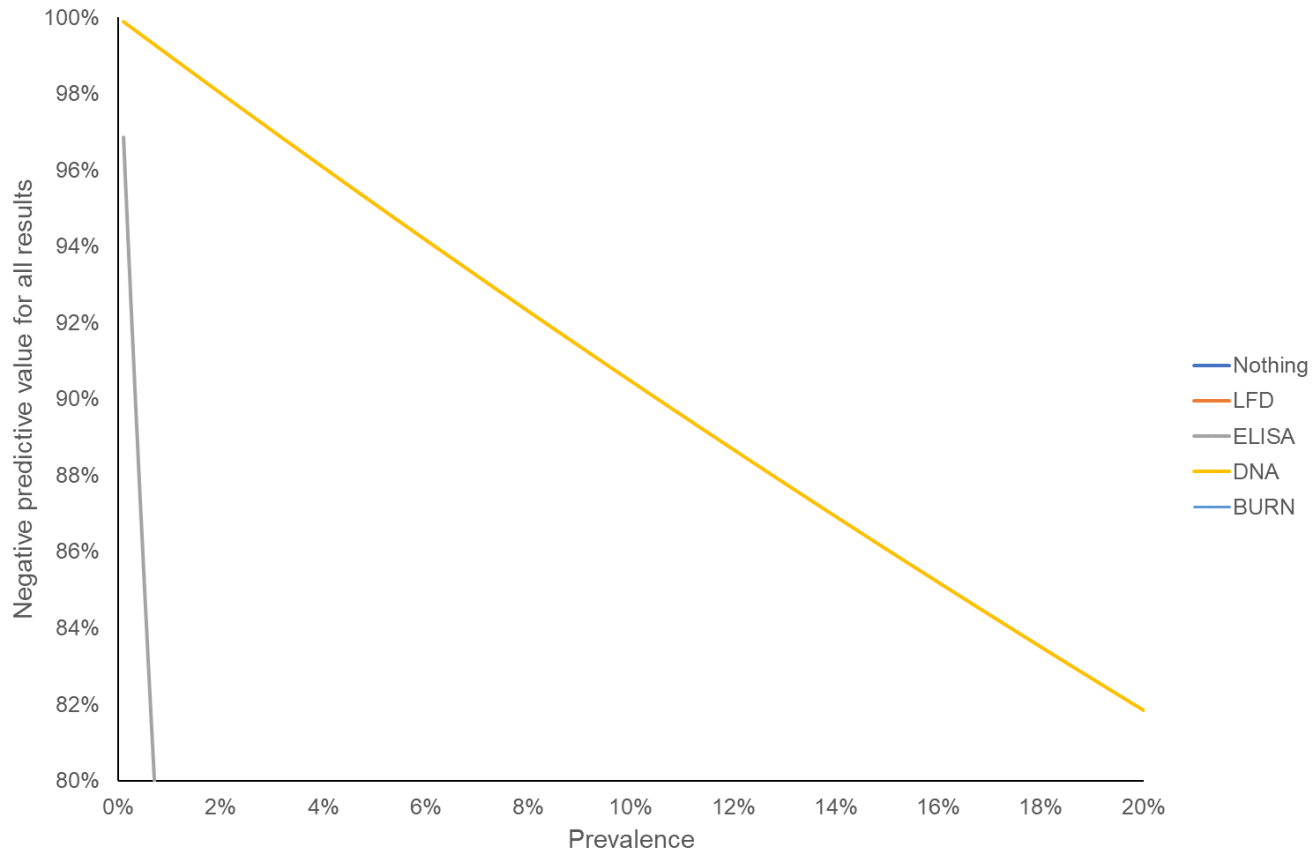
$$N = \frac{B}{L + F \cdot (1 - Sp)} \quad NPR = \frac{Sp}{1 - Se}$$

Eradication



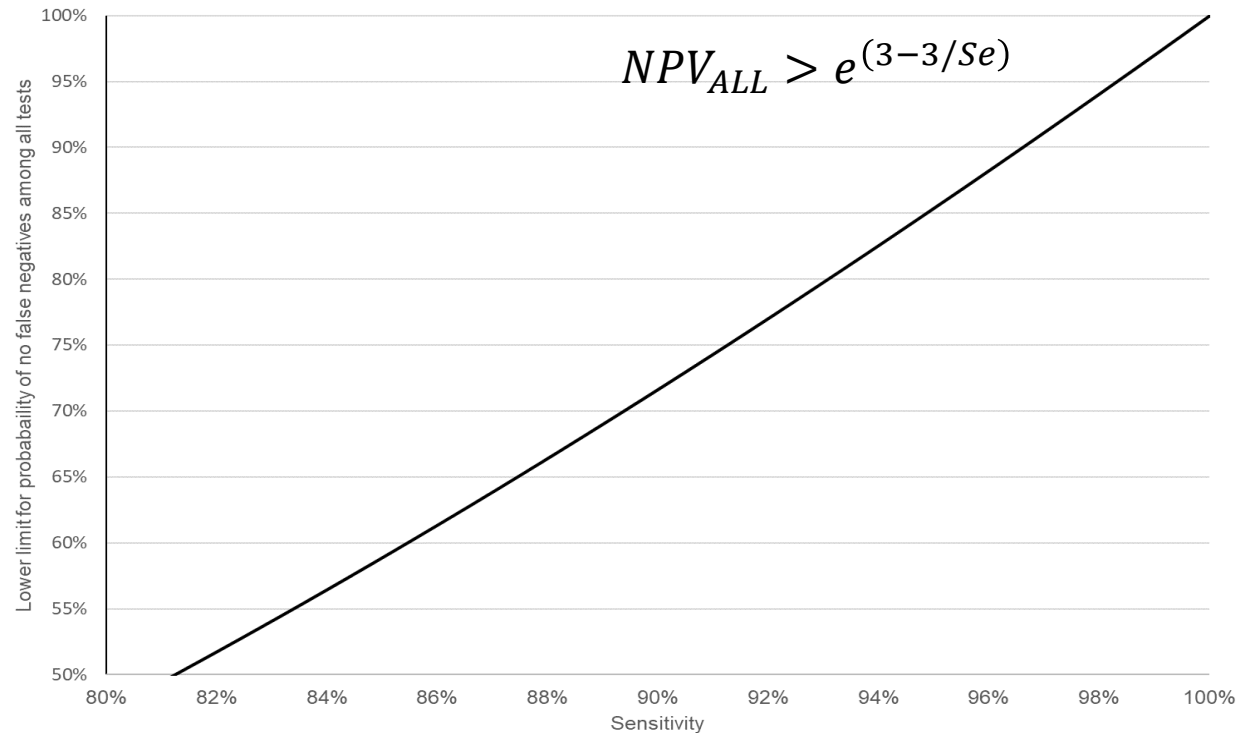
$$N = \frac{B}{L+F.(1-Sp)} \quad NPV = \frac{(1-prev).Sp}{(1-prev).Sp+prev.(1-Se)}$$

Eradication II



$$N = \frac{B}{L + F \cdot (1 - Sp)} \quad NPV_{All} = \left(\frac{(1 - prev) \cdot Sp}{(1 - prev) \cdot Sp + prev \cdot (1 - Se)} \right)^{\text{number neg}}$$

Lower limit for probability of no false negatives where all tests are negative

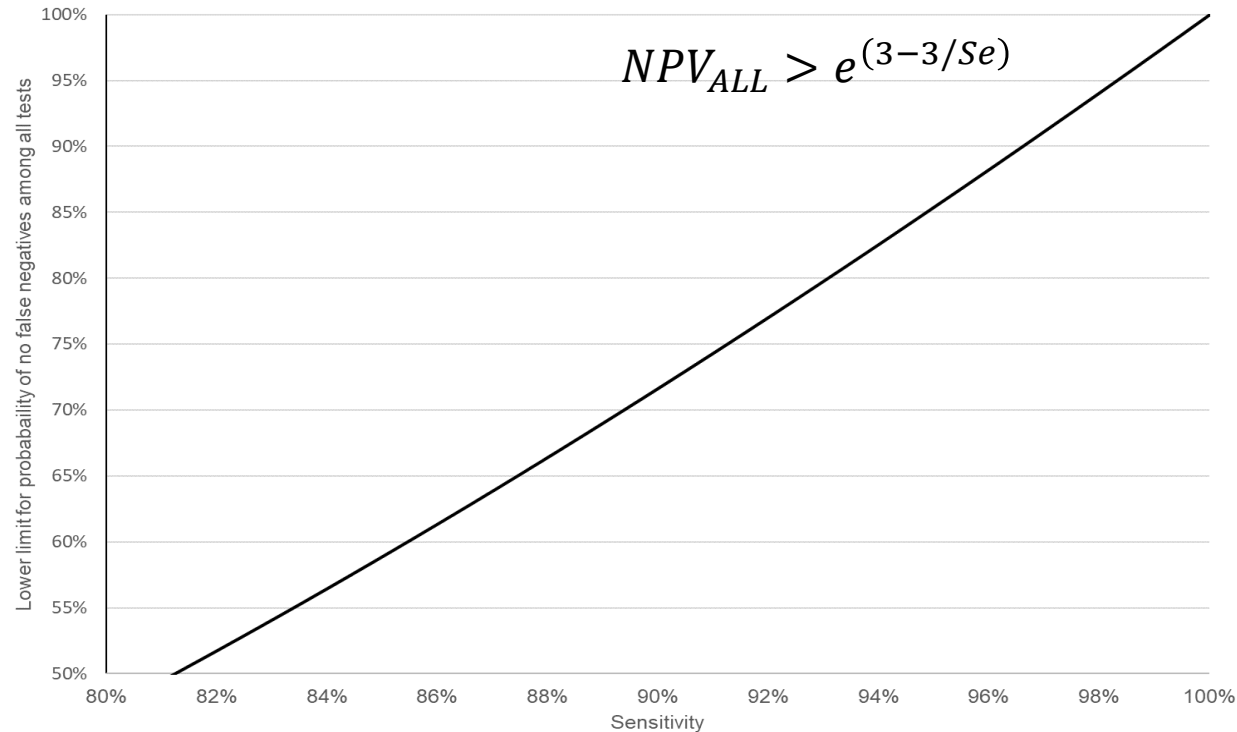


Approximately: $NPV_{ALL} > 1 - 3 \cdot (1 - Se)$

Where $Se > 0.85$

Lower limit for probability of no false negatives where all tests are negative

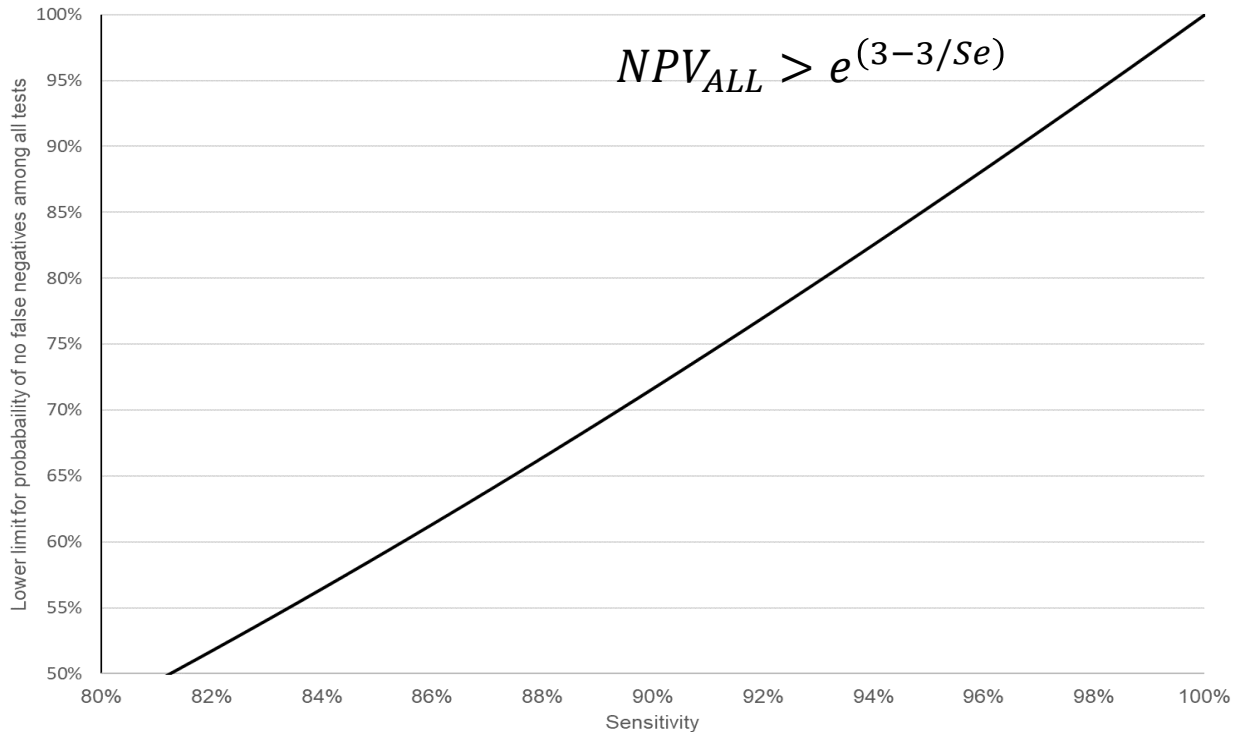
$$NPV_{ALL} > 95\% \text{ where } Se > 98.4\%$$



Approximately: $NPV_{ALL} > 1 - 3 \cdot (1 - Se)$

Where $Se > 0.85$

Lower limit for probability of no false negatives where all tests are negative



$$NPV_{ALL} > 95\% \text{ where } Se > 98.4\%$$

A method showing no false negatives in 188 representative tests has sensitivity $> 98.4\%$

IF you want to use a test to give high assurance about ALL tested samples $Se > 98.4\%$ is a reasonable requirement

If you start seeing positives then you can't use testing alone to COMPLETELY separate all infected individuals from uninfected individuals in that population with high confidence

Approximately: $NPV_{ALL} > 1 - 3 \cdot (1 - Se)$

Where $Se > 0.85$

Monitoring prevalence

- Several potential measures of measurement performance: e.g. standard error of estimates at a prevalence; width of confidence interval associated with estimates; power to detect difference between two different prevalences.
- Here we will use the prevalence package in R to estimate the width of a 95% confidence interval where the central estimate of prevalence is 1% and 5%

Monitoring prevalence

- Testing budget £10 000; no retesting of positive results

Test	Cost (L)	Performance		N
		Sensitivity (Se)	Specificity (Sp)	
LFD	10	50%	99.5%	1000
ELISA	30	90%	99.7%	333
DNA	100	99%	100%	100.

Monitoring prevalence

- Testing budget £10 000; no retesting of positive results

Test	Cost (L)	Performance		N	Pos.	Central estimate 1%		
		Sensitivity (Se)	Specificity (Sp)			Est	95% C.I.	
LFD	10	50%	99.5%	1000	9	0.9%	0.1%	2.5%
ELISA	30	90%	99.7%	333	4	1.0%	0.2%	3.0%
DNA	100	99%	100%	100	1	1.1%	0.2%	5.5%

```
>truePrev(x=9,n=1000,SE=0.5,SP=0.995)
  mean median  mode    sd 2.5% 97.5%
TP 0.010  0.010 0.009 0.006 0.001 0.025
```

Monitoring prevalence

- Testing budget £10 000; no retesting of positive results

Test	Cost (L)	Performance		N	Pos.	Central estimate 5%		
		Sensitivity (Se)	Specificity (Sp)			Est	95% C.I.	
LFD	10	50%	99.5%	1000	30	5.0%	3.2%	7.6%
ELISA	30	90%	99.7%	333	16	5.0%	3.0%	8.2%
DNA	100	99%	100%	100	5	5.1%	2.2%	11.1%

Monitoring prevalence

- Fixed number of tests: effect of test performance only

Test	Cost (L)	Performance		N	Pos.	Central estimate 1%		
		Sensitivity (Se)	Specificity (Sp)			Est	95% C.I.	
LFD	10	50%	99.5%	1000	9	0.9%	0.1%	2.5%
ELISA	30	90%	99.7%	1000	11	0.9%	0.4%	1.8%
DNA	100	99%	100%	1000	10	1.1%	0.6%	1.8%

Test	Cost (L)	Performance		N	Pos.	Central estimate 5%		
		Sensitivity (Se)	Specificity (Sp)			Est	95% C.I.	
LFD	10	50%	99.5%	1000	30	5.0%	3.2%	7.6%
ELISA	30	90%	99.7%	1000	48	5.0%	3.7%	6.7%
DNA	100	99%	100%	1000	50	5.0%	3.9%	6.6%

Using TPS results to assess fitness for purpose

- Simple approximate formulas can be used to estimate cost-performance for early detection, delineation and tasks requiring confidence about individual results such as eradication
- Assessing cost performance of tests with imperfect sensitivity and specificity for estimating prevalence needs more advanced methods BUT! there are easy-to-use tools available

Observations on test performance and fitness for purpose

- Early detection and delineation can be achieved most quickly with low sensitivity methods if they are cheap enough
- Where we require high confidence about each individual result this can occasionally be achieved with high sensitivity methods
- High confidence about ALL negative test results can only be achieved with high sensitivity methods AND where all results are negative
- With the right statistical tools methods with low sensitivity and imperfect specificity can provide accurate estimates of prevalence